INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

INPP @ OHIO UNIVERSITY

Nuclear Interactions

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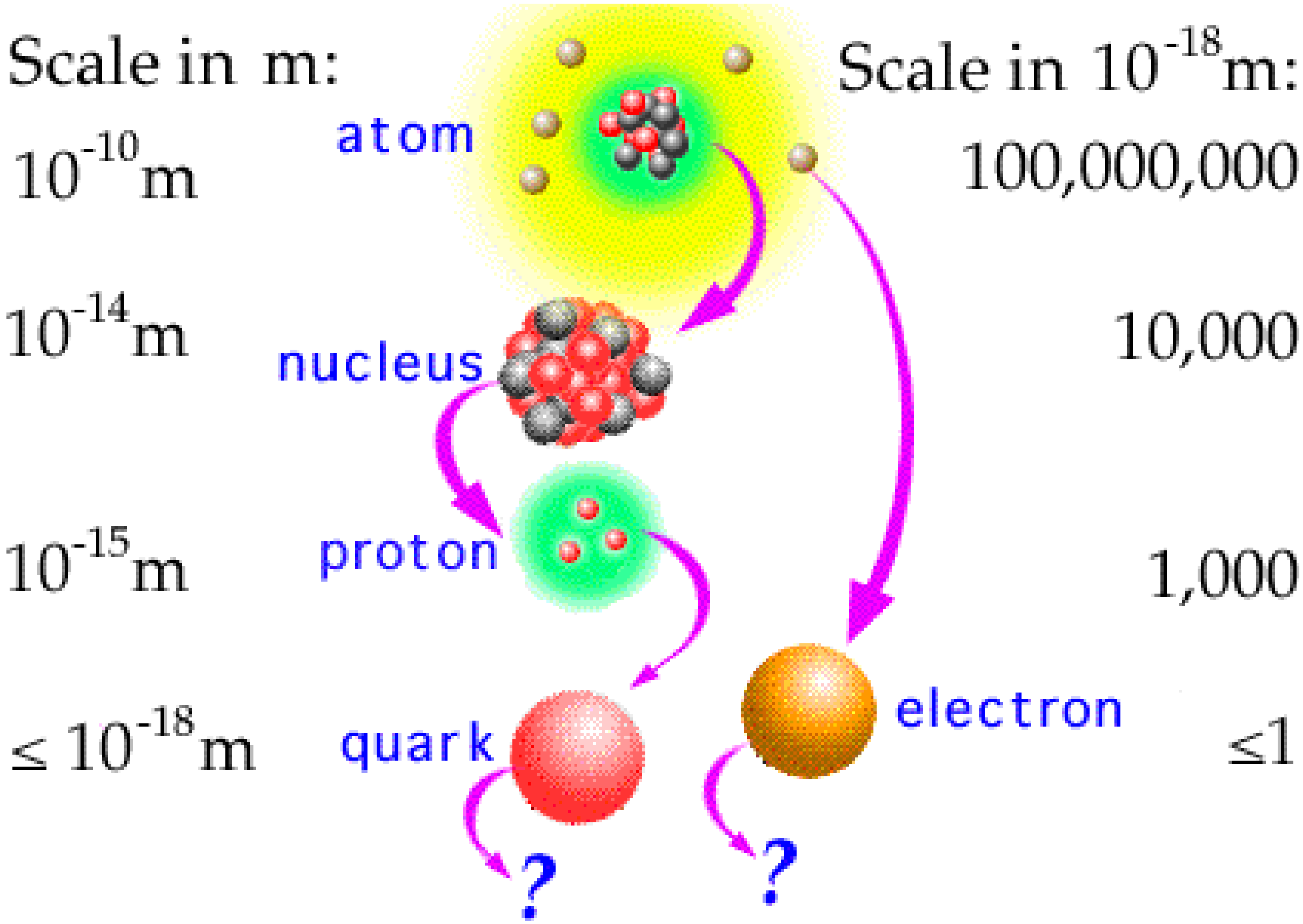
1/14/2009

Supported by: U.S. DOE, NERSC, OSC

Scales in Nature



Energy Densities:

- Spring under tension
- Binding energy of electrons in a heavy atom
- Binding energy of nucleons

- $\approx 10^6$ J/m^3
- $\approx 10^{16}$ J/m^3 [eV]
- $\approx 10^{32}$ J/m^3 [MeV]

Nuclear force → strong interaction

Travel through History:

1930's:

- Chadwick (1932) discovers the neutron
- Heisenberg (1932) proposes the first phenomenology (Isospin)
- Yukawa (1935) proposes the pion and his Meson Hypothesis

1940's:

- Discovery of the pion in cosmic rays (1947)
- and in the Berkeley Cyclotron Lab (1948)
- Nobel Prize awarded to Yukawa (1949)
- Rabi (1948) measures the quadrupole moment of the deuteron

Low Energy Nuclear Physics
Smallest Nucleus: Deuteron (np)

Ground State Properties	Value
Binding Energy	2.22457312(22) MeV
Spin & Parity	1+
Iso Spin (n,p)	0
Magnetic Dipole Moment	0.857406(1) μ_N
Electric Quadrupole Moment	0.28590(3) fm^2
Radius	1.963(4) fm

Nuclear Force has Tensor Component

- Yukawa 1934:
  - postulated π as carrier of the nuclear force

$$-\frac{\vec{\sigma}_1 \cdot \vec{p} \;\; \vec{\sigma}_2 \cdot \vec{p}}{m_\pi^2 + p^2} \approx \left\{ \frac{3}{r^2} \left(\vec{\sigma}_1 \cdot \vec{r} \;\; \vec{\sigma}_2 \cdot \vec{r} \right) - \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \right\} \frac{e^{-m_\pi r}}{r}$$

- Analogy EM: Quadrupole momentum tensor:

$$Q_{ij} = \int \rho(x)(3x_i x_j - r^2 \delta_{ij})\, d^3x \ ,$$

- Consequence: $[H,L_3] \neq 0$ but $[H,J_3]=0$

More experimental evidence

- Comparison of the binding energies of:
  - 2H (deuteron), 3H (triton), 4He (alpha - particle)
  - show that the nuclear force is of finite range (1 − 2 fm) and very strong within that range.
- For nuclei with A > 4:
  - the energy saturates: Volume and binding energies of nuclei are proportional to the mass number A.
  - Binding energy of all nuclei ~ $A^{1/3}$
  - Nuclear force must be repulsive at short distances
- Spin-orbit force:
  - Observation of large polarizations of scattered nucleons perpendicular to the plane of scattering.

Travel through History:

1950's:

- Taketani, Nakamura, Sasaki (1951):
  - One Pion Exchange (OPE)   o.k.
  - Multi-Pion Exchanges – problems!
- Taketani, Machida, Onuma (1952): multi-pion exchanges
  - Concept of renormalization was not worked out
- Gammel-Thaler (1957): First NN potential model
- Okubo-Marshak (1958) [Ann.Phys. 4, 166]
  - General consideration about the form of an NN interaction using only invariance properties of a Hamiltonian.

Travel through History:

1960's:

- Many pions = multi-pion resonances:
  - $\sigma(600)$, $\rho(770)$, $\omega(782)$
  - Input from High Energy Physics
- One-Boson-Exchange Models
  - General Invariance structure given by Poincaré Invariance.
- Refined phenomenology: Reid Potential (1968)

1970's and 1980's:

- Sophisticated models for two-pion exchange
  - Paris Potential (Lacombe et al., Phys. Rev. C 21, 861 (1980)
  - Bonn Potential (Machleidt, Holinde, Elster, Phys. Rep. 149, 1 (1987))
- Quark Cluster Models
- Begin of Effective Field Theory studies

Travel through History:

1990's:

- "High-Precision" NN Potentials:
  - Nijmegen I, II, '93, Reid93 (Stoks et al. 1994
  - Argonne V18 (Wiringa et al, 1995)
  - CD-Bonn (Machleidt et al. 1996 and 2001)
- Advances in Effective Field Theory – Chiral Perturbation Theory
  - Weinberg (1990)
  - Ordonez, Ray, van Kolck (1996) …

2000 + :

- Multi-Pion theory constrained by Chiral Symmetry:
  - Chiral EFT (lots of ongoing work)
- 2006: Nucleon-nucleon interaction from Lattice QCD,
  final confirmation of meson hypothesis of Yukawa?

Okubo – Marshak Invariants

NN Hamiltonian: $$H = \frac{1}{2m}\left(p_1^2 + p_2^2\right) + V_{NN}(1,2)$$

$$V_{NN}(1,2) \equiv V_{NN}\left(\vec{r}, \vec{R}, \vec{p}, \vec{P}, \vec{\sigma}^{(1)}, \vec{\sigma}^{(2)}, \vec{\tau}^{(1)}, \vec{\tau}^{(2)}\right)$$ (V_{NN} Hermitian)

- Translation invariance
- Rotation invariance
- Space reflection invariance
- Time reversal invariance
- Galilean invariance
- Invariance under the exchange of nucleons 1 and 2
- Almost Isospin Symmetry

(www.phy.ohiou.edu/~elster/lectures/fewblect_1.pdf Section 1.9)

Most general, velocity-dependent, non-relativistic NN potential

$$V_{NN} \equiv V_{NN}(\vec{r}, \vec{R}, \vec{p}, \vec{P}, \vec{\sigma}^{(1)}, \vec{\sigma}^{(2)}, [\vec{\tau}^{(1)} \cdot \vec{\tau}^{(2)}])$$

$$= V_1(\vec{r}, \vec{R}, \vec{p}, \vec{P}, \vec{\sigma}^{(1)}, \vec{\sigma}^{(2)}) + (\vec{\tau}^{(1)} \cdot \vec{\tau}^{(2)}) \, V_2(\vec{r}, \vec{R}, \vec{p}, \vec{P}, \vec{\sigma}^{(1)}, \vec{\sigma}^{(2)})$$

with

$$\begin{aligned} V_i &= V_i^c(r^2, p^2, L^2) + \vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)} \, V_i^\sigma(r^2, p^2, L^2) \\ &+ S_{12} \, V_i^T(r^2, p^2, L^2) + \vec{S} \cdot \vec{L} \, V_i^{LS}(r^2, p^2, L^2) \\ &+ [(\vec{\sigma}^{(1)} \cdot \vec{L})(\vec{\sigma}^{(2)} \cdot \vec{L}) + (\vec{\sigma}^{(2)} \cdot \vec{L})(\vec{\sigma}^{(1)} \cdot \vec{L})] \, V_i^{\sigma L}(r^2, p^2, L^2) \end{aligned}$$

and $$S_{12} = \frac{3}{r^2} (\vec{\sigma}^{(1)} \cdot \vec{r})(\vec{\sigma}^{(2)} \cdot \vec{r}) - (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)})$$

e.g. one-pion-exchange (OPE):

$$V_{OPE}(r) \equiv \frac{1}{12}\frac{g_{ps}^2}{4\pi} m_{ps}\left[\left(\frac{m_{ps}}{m}\right)^2 \mathcal{Y}(m_{ps}r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + Z(m_{ps}r)\,S_{12}\right]$$

$$\mathcal{Y}(x) = e^{-x}/x$$

$$Z(x) = (m_\alpha/m)^2(1 + 3/x + 3/x^2)\mathcal{Y}(x)$$

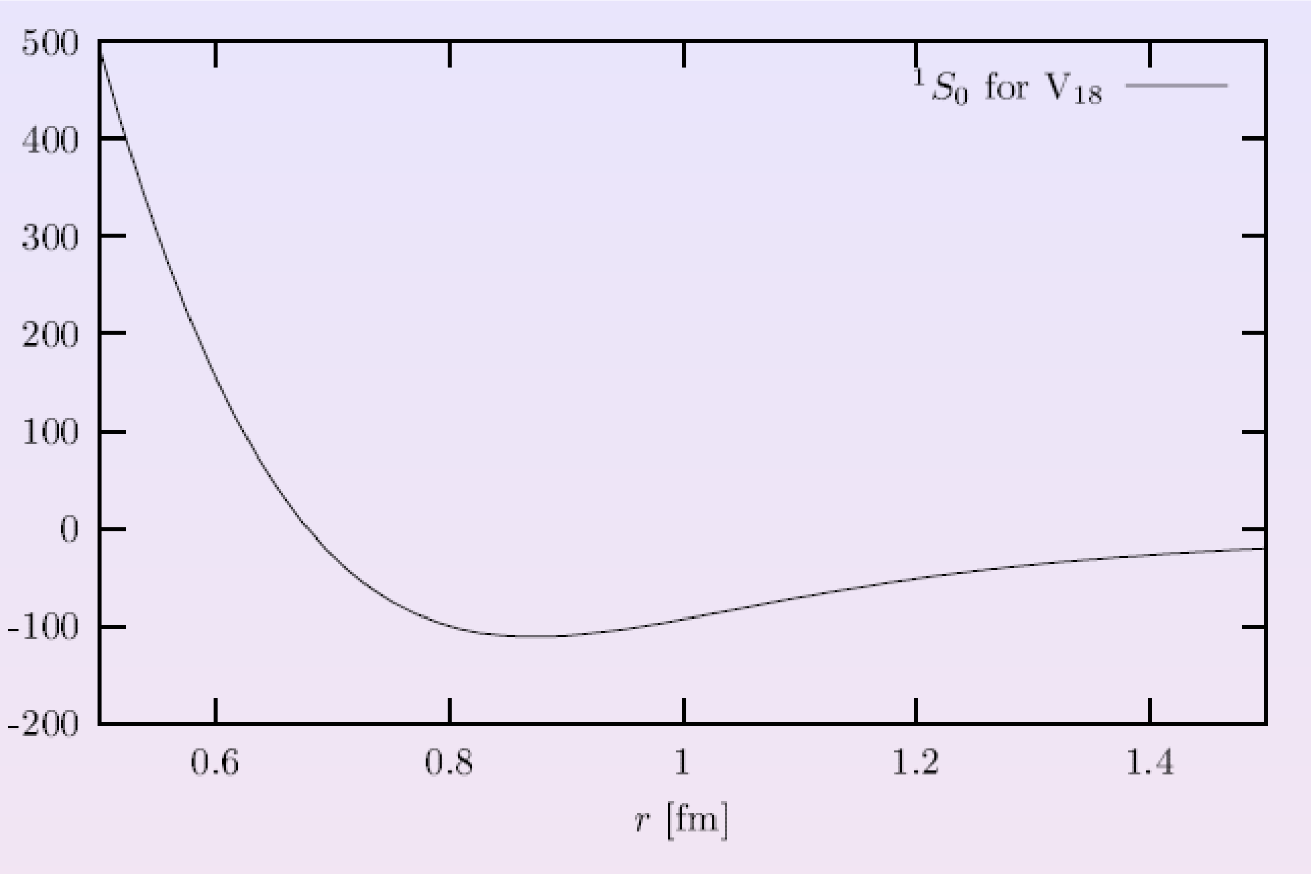
Momentum space:

$$V_{OPE}(k) = -\frac{g_{ps}^2}{4m^2}\frac{(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k})}{k^2+m_{ps}^2}$$

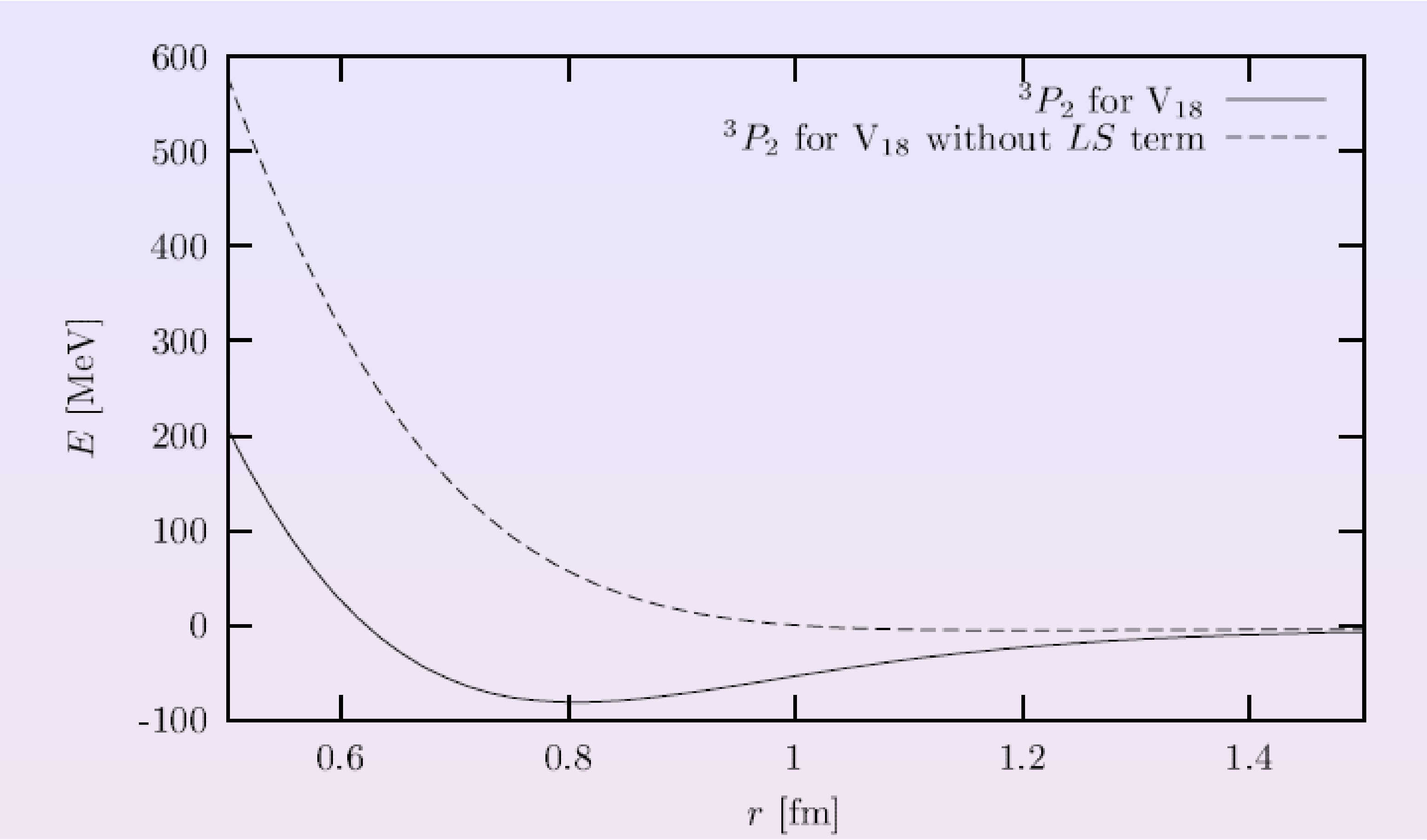
Task:
Derive / obtain functions $V_i^{\alpha}(r^2, p^2, L^2)$:

- **Ansatz I** (based on Okubo-Marshak Invariants)
  - Functions: Yukawa type ~ const $*$ $\exp(-m_\alpha r)/r$
  - const $\equiv$ magnitue of term; $m_\alpha \equiv$ range of term
  - For longest range: OPE
  - Choose superposition of terms with different strengths and ranges and fit to NN data
  - NN data:
    - cross section, polarization and double polarizations
  - Potentials:
    - Reid (1968)
    - Nijmegen potentials
    - Argonne V18

Potential projected on angular momentum eigenstates



Notation: $^{2S+1}L_J$



$^3P_2$ for V_{18}
$^3P_2$ for V_{18} without LS term
600
500
400
300
200
100
0
-100
E [MeV]
0.6
0.8
1
1.2
1.4
r [fm]

Ansatz II

- Start from relativistic Lagrangians
  - Invariants: bilinear covariants (5 quantities invariant under Lorentz group)
- Input: mesons from the particle data (mesons $\equiv \langle q\bar{q} \rangle$ pairs)
  - Pseudo-scalar / scalar / vector mesons
- Functional forms of the V_i^α (r^2, p^2, L^2) given
  - From evaluation of Feynman diagrams with exchanged meson
- Parameters
  - Coupling constants, cutoff parameter(s)
- One-boson-exchange (OBE) potentials
  - Bonn series (A,B,C)
  - Utrecht models
  - Gross model
  - CD-Bonn (2001)

Dramatis Personae:

Mesons	Mass (MeV)
π	138.03
η	548.8
σ	≈ 550.0
ρ	770
ω	782.6
δ	983.0
K	495.8
$K^\star$	895.0

Phenomenological Lagrangians for spin ½ baryons:

$$\mathcal{L}_{ps} = g^{ps}\overline{\Psi}\gamma^5\Psi\phi^{(ps)},$$

$$\mathcal{L}_s = g^s\overline{\Psi}\Psi\phi^{(s)},$$

$$\mathcal{L}_{pv} = \frac{f_\pi}{m_\pi}\overline{\psi}\gamma_5\gamma_\mu\psi\partial^\mu\phi_\pi.$$

$$\mathcal{L}_v = g^v\overline{\Psi}\gamma_\mu\Psi\phi^{(v)}_\mu + g^t\overline{\Psi}\sigma^{\mu\nu}\Psi\left(\partial_\mu\phi^{(v)}_\nu - \partial_\nu\phi^{(v)}_\mu\right)$$

Ansatz II cont'd

- Scalar meson σ responsible for attractive piece in NN force
  - Existence questionable in the history of PDG
- Explicit 2-pion exchange into OBE models
  - Via Dispersion relations
    - Paris Potential
  - Via explicit incorporation of explicit two-meson exchanges ($\pi\pi$, $\pi\rho$, $\rho\rho$) and the Delta resonance
    - Bonn Potential (1987)

Assessment:

Potentials based on Galilei (or Poincaré) invariance and meson exchange:

- Very successful
  - Perfect description of world NN data base
- However:
  - Little relation to QCD
  - Model dependence
  - Theoretical uncertainties impossible to

… mostly effective
but
not systematic ..

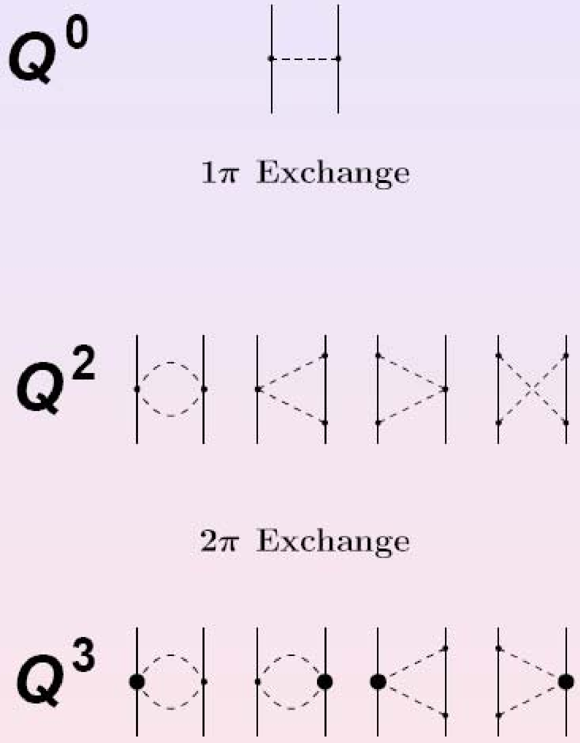
Ansatz III : Chiral Effective Field Theory

- Write down the most general Lagrangian including all terms consistent with the assumed symmetries, in particular the spontaneously broken chiral symmetry
- Calculate all Feynman diagrams
  - There will be infinitely many
- Find a scheme for assessing the importance of the various diagrams
- Organize the contributions in terms of $\Lambda_\chi \approx 1$ GeV

$$\mathcal{L}_{NN} = \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN}^{(4)} + \ldots,$$

where the superscript refers to the number of derivatives or pion mass insertions (chiral dimension). Good review: Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006).

Chiral EFT



χPT

- The most important irreducible one- and two-pion exchange contributions to the NN interaction up to order Q^3.
- Vertices denoted by small dots are from $\widehat{\mathcal{L}}^{(1)}_{\pi N}$.
- Large dots refer to $\widehat{\mathcal{L}}^{(2)}_{\pi N,\,\mathrm{ct}}$

Major advantage: Systematic expansion **and** error estimates in each order