The Giant Monopole Resonance in the Sn Isotopes: Why is Tin so “Fluffy”?

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The isoscalar giant monopole resonance (GMR) has been investigated in a series of Sn isotopes (A=112–124) using inelastic scattering of 400-MeV α particles at extremely forward angles (including 0°). The primary aim of the investigation has been to explore the role of the “symmetry-energy” term in the expression for nuclear incompressibility. It is found that the excitation energies of the GMR in the Sn isotopes are significantly lower than those expected from the nuclear incompressibility previously extracted from the available data on the compressional-mode giant resonances.

The investigation of the compressional-mode giant resonances—the Isoscalar Giant Monopole Resonance (GMR) and the Isoscalar Giant Dipole Resonance (ISGDR), an exotic compressional mode of nuclear oscillation—continues to remain an active area of work and interest. The primary motivation for the investigation of these modes is that they provide a direct experimental determination of the incompressibility of infinite nuclear matter, K_{∞}, a quantity of critical importance to understanding the nuclear equation of state.

Experimental identification of these two modes requires inelastic scattering measurements at extremely-forward angles (including 0°, where the GMR Cross sections are maximal). Recent experimental work, using inelastic scattering of α particles, has been carried
out at RCNP, Osaka (400 MeV) [1–8], at Texas A & M University (240 MeV) [9–14], and at KVI, Groningen (200 MeV) [15]. This has been synergistically enhanced by contemporaneous theoretical work by several groups [16–39]; some of the theory work was previously reviewed by Colò [19] and has been updated for this Conference [21].

It is now generally accepted that the best method to extract the nuclear incompressibility, \( K_{\infty} \), from the compressional-mode giant resonances, first proposed by Blaizot [40], is to compare the experimental GMR energies with the theoretical values obtained from RPA calculations using different established interactions; the \( K_{\infty} \) associated with the interaction that best reproduces the GMR (and ISGDR) energies, is considered the “correct” experimental value. Based on this procedure, it has been established [1,4,7,19,20] that both the compressional-mode giant resonances are consistent with \( K_{\infty} \sim 230 \text{ MeV} \). However, when we started the measurements reported here, the relativistic [24,26,31] and non-relativistic calculations [19,16,33] led to significantly different values for \( K_{\infty} \) from the same GMR data, the values from non-relativistic calculations being, typically, 210–230 MeV and for the relativistic calculations 250–270 MeV. [Efforts have since been made to create interactions within the two approaches that are mutually consistent in terms of the value of the nuclear incompressibility employed [20,29,35].] This “disagreement” has engendered a healthy debate, and it appeared that the primary difference between the non-relativistic and non-relativistic calculations pertained to the values of symmetry energy term employed; however, the experimental data available prior to the work reported here did not provide sufficient sensitivity to distinguish between the two.

The excitation energy of the GMR is expressed in the scaling model [41] as:

\[
E_{\text{GMR}} = \hbar \left( \frac{K_A}{m} \right) \sqrt{\frac{\langle r^2 \rangle}{m}}
\]

where \( K_A \) is the incompressibility of the nucleus and can be expressed as:

\[
K_A \approx K_{\infty}(1 + cA^{-1/3}) + K_\tau((N - Z)/A)^2 + K_{\text{Coul}}Z^2A^{-4/3}.
\]

Here \( K_\tau \) and \( K_{\text{Coul}} \) are negative quantities. Of these, \( K_{\text{Coul}} \) is, basically, model independent and the coefficient “\( c \)” was found to be close to -1 in both relativistic and non-relativistic models. That leaves \( K_\tau \) and a more negative value for this quantity leads to extracting from the experimental \( K_A \) values a larger value for the \( K_{\infty} \) [20,27]. Indeed, the typical values for \( K_\tau \) are \( \sim -300 \text{ MeV} \) and \( \sim -700 \text{ MeV} \) for the “standard” non-relativistic and relativistic models, respectively.

The best place to observe a direct effect of this difference in \( K_\tau \) values is in a series of isotopes where the factor \( ((N - Z)/A) \) would vary significantly without affecting the other terms in Eq. 2 in any substantial way. The Sn isotopes afford such an opportunity. Between \(^{112}\text{Sn}\) and \(^{124}\text{Sn}\), this factor increases by \( \sim 80\% \) and it was estimated, for example, that the change in the GMR-energy in going from \(^{112}\text{Sn}\) to \(^{124}\text{Sn}\) would be different in the two calculations by \( \sim 0.5 \text{ MeV} \).

We have measured the GMR strength distributions in the Sn isotopes (\( A=112,114,116,118,120,122,124 \)) using inelastic scattering of 400-MeV \( \alpha \) particles. The experiments were carried out at the Research Center for Nuclear Physics (RCNP), Osaka University. Details of the experimental procedures have been provided previously [4,5]. Single-turn-extraction \( \alpha \) beams from the Ring Cyclotron were transported to the target chamber
of the Grand Raiden spectrometer [42] via a beam-analyzing system without use of any slits. Inelastically scattered particles were momentum-analyzed in the spectrometer. The focal-plane detector system was comprised of 2 MWDC’s and 2 scintillators, providing both horizontal and vertical positions of incoming particles. The scattering angle at the target could be determined from ray-tracing. The primary beam was stopped in three different Faraday cups, depending on the angle-setting of the spectrometer [5]. Inelastic scattering data were taken over the angular range $0^\circ$–$9^\circ$. In addition, elastic scattering data were obtained (over the range $3^\circ$–$25^\circ$) on all targets to obtain suitable optical-model parameters for each nucleus under investigation. Calibration data were obtained using $^{12}$C and $^{24}$Mg targets; a CH$_2$ target was employed to account for the hydrogen-contamination in some of the targets.

A unique aspect of the data obtained in our measurements has been the complete elimination of all “instrumental” background [1,5]. This has been possible because of the ion-optics of Grand Raiden: The particles scattered from the target position are focused in the vertical direction at the focal plane while those that undergo a change in trajectory (because of scattering off the slits, or the wall or yoke of the spectrometer, for example) are defocused. As a result, the “true” scattering events can be separated from the “in-

![Figure 1](image-url) Inelastic $\alpha$ scattering spectra at “0°” for the various Sn targets measured in the present work. The average angle after accounting for the non-circular shape of the opening of the spectrometer is 0.69°.
strumental background.” This allows us to carry out further analysis without the need for arbitrarily subtracting a background from the spectra; the uncertainties associated with such background-subtraction procedures had been the bane of giant resonance analyses and a point of constant criticism. In our analysis, we treat the continuum as composed of a combination of higher multipoles and include it in the multipole analysis of the inelastic scattering spectra to extract the strength-distributions of various multipoles. The multipole-decomposition procedure is similar to those used previously by Bonín et al. [43] and by Clark et al. [11]; details have been provided elsewhere [3–5,7].

The “0°” inelastic spectra for the Sn isotopes are presented in Fig. 1. In all cases, the spectrum is dominated by the GMR peak near $E_x \sim 15$ MeV. The GMR strengths extracted from the multipole-decomposition analysis are shown in Fig. 2. The solid lines in Fig. 2 represent Lorentzian fits to the observed strength distributions. The choice of the Lorentzian shape is arbitrary; the final results are not affected in any significant way by using a Gaussian shape instead. The extracted GMR parameters, including the various moment-ratios, are presented in Table 1.

Using Eq. (1) and the extracted moment ratios $m_1/m_0$, we have obtained the values of

![Figure 2](https://example.com/figure2.png)

Figure 2. Extracted GMR strength distributions in $^{112−124}$Sn from the multipole-decomposition analysis. The solid lines show results of Lorentzian fits to the data.
Table 1
Lorentzian-fit parameters and various moment-ratios for the GMR strength distributions in the Sn isotopes, as extracted from multipole-decomposition analysis in the present work.

<table>
<thead>
<tr>
<th>target</th>
<th>$E_0$ (MeV)</th>
<th>$\tau$ (MeV)</th>
<th>$m_1/m_0$ (MeV)</th>
<th>$\sqrt{m_3/m_1}$ (MeV)</th>
<th>$\sqrt{m_1/m_{-1}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{112}$Sn</td>
<td>16.1 ± 0.09</td>
<td>4.0 ± 0.42</td>
<td>16.2 ± 0.13</td>
<td>16.7 ± 0.15</td>
<td>16.1 ± 0.12</td>
</tr>
<tr>
<td>$^{114}$Sn</td>
<td>15.9 ± 0.14</td>
<td>4.1 ± 0.38</td>
<td>16.1 ± 0.12</td>
<td>16.5 ± 0.17</td>
<td>15.9 ± 0.11</td>
</tr>
<tr>
<td>$^{116}$Sn</td>
<td>15.8 ± 0.13</td>
<td>4.1 ± 0.33</td>
<td>15.8 ± 0.10</td>
<td>16.3 ± 0.16</td>
<td>15.7 ± 0.12</td>
</tr>
<tr>
<td>$^{118}$Sn</td>
<td>15.6 ± 0.08</td>
<td>4.3 ± 0.38</td>
<td>15.8 ± 0.11</td>
<td>16.3 ± 0.14</td>
<td>15.6 ± 0.13</td>
</tr>
<tr>
<td>$^{120}$Sn</td>
<td>15.4 ± 0.19</td>
<td>4.9 ± 0.54</td>
<td>15.7 ± 0.10</td>
<td>16.2 ± 0.15</td>
<td>15.5 ± 0.11</td>
</tr>
<tr>
<td>$^{122}$Sn</td>
<td>15.0 ± 0.16</td>
<td>4.4 ± 0.41</td>
<td>15.4 ± 0.10</td>
<td>15.9 ± 0.18</td>
<td>15.2 ± 0.11</td>
</tr>
<tr>
<td>$^{124}$Sn</td>
<td>14.8 ± 0.21</td>
<td>4.5 ± 0.52</td>
<td>15.3 ± 0.10</td>
<td>15.8 ± 0.14</td>
<td>15.1 ± 0.11</td>
</tr>
</tbody>
</table>

Figure 3. Systematics of $K_A$ values obtained from the moment ratios $m_1/m_0$ for the GMR strength distributions in the Sn isotopes as a function of the “symmetry-parameter” ($N-Z)/A$ (squares). A least-squared quadratic fit to the data is shown as a solid line; the parameters of the fit are shown in the inset.

$K_A$ for the Sn isotopes. These are presented in Fig. 3 as a function of the “symmetry-parameter” (($N-Z)/A$). A reasonable approximation of Eq. (2) is that $K_A$ has a quadratic relationship with this “symmetry-parameter” (of the type $C = A + Bx^2$, with the coeffi-
cient “B” being the parameter $K_r$). A least-squared quadratic fit to the data is also shown in the figure. The fit gives a value of $K_r = -395 \pm 40$ MeV. While, admittedly, this value would have a larger uncertainty if one accounts for the use of the simplified quadratic equation, and for all possible systematic effects, it would appear that the symmetry-energy term used in the non-relativistic calculations ($\sim -300$ MeV) is closer to the experimental value than that used in the relativistic calculations ($\sim -700$ MeV).

The moment-ratios, $m_1/m_0$ for the GMR strengths in the Sn isotopes are shown in Fig. 4 and compared with recent calculations from Colò [44] and Piekarewicz [45]. The interactions used in these calculations are those that very closely reproduce the GMR energies in $^{208}$Pb and $^{90}$Zr. But, clearly, the predicted GMR energies for all Sn isotopes studied in this work are significantly larger than the experimentally observed values. Fig. 4

Figure 4. Systematics of the moment ratios $m_1/m_0$ for the GMR strength distributions in the Sn isotopes as a function of mass number (filled squares). The experimental results are compared with results of very recent calculations by Colò [44] (filled circles) and Piekarewicz [45] (triangles). Also shown are the moment ratios for $^{112}$Sn and $^{124}$Sn, reported by the Texas A & M group [13](inverse triangles).
also shows the GMR energies extracted for $^{112}\text{Sn}$ and $^{124}\text{Sn}$ in recent Texas A & M work [13]; the “agreement” with those is even worse! This leads directly to the question posed in the title of this report: Why are the Tin isotopes so “fluffy”? Are there any nuclear structure effects that need to be taken into account to describe the GMR energies in the Sn isotopes? Or, more provocatively, do the GMR energies depend on something more than the nuclear incompressibility, requiring a modification of the scaling relationship given in Eq. (1)? In the latter case, why does this “effect” show up only in the Sn isotopes? This remains a challenge to the theoretical calculations describing the GMR.

To summarize, the isoscalar giant monopole resonance (GMR) has been investigated in a series of Sn isotopes (A=112–124), using “small-angle” inelastic scattering of 400-MeV $\alpha$ particles. The primary aim of these measurements has been to explore the effect of the “symmetry-energy” term, $K_\tau$, in the expression for nuclear incompressibility. The preliminary value of $K_\tau$ extracted from these measurements ($K_\tau = -395 \pm 40$ MeV) is not too different from the typical values ($\sim -300$ MeV) employed in the non-relativistic calculations. It is found that the experimental GMR energies are significantly lower than those predicted by recent non-relativistic and relativistic calculations, leaving a challenge for the theories.

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